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A COMPARISON OF TWO TESTS FOR THE SIGNIFICANCE OF A MEAN VECTOR

by

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ABSTRACT

Hotelling's T^2 procedure for testing the significance of a mean vector treats all variables symmetrically and may not be appropriate if the variables carry unequal importance. The step-down procedure due to J. Roy (1958), a possibly appropriate alternative, is studied in this paper. An underlying invariance structure is established and then used to develop a canonical form for studying the power functions of the two methods. A Monte Carlo experiment is conducted in this framework and conclusions are reported.

Key Words: Hotelling's T^2, Step-down procedure, invariance, empirical power.

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A COMPARISON OF TWO TESTS FOR THE SIGNIFICANCE OF A MEAN VECTOR

1. Introduction

The problem of testing the significance of the mean of a random vector or the equivalent problem of comparing the means of two random vectors arises in various statistical contexts. The best known solution to the problem is based upon Hotelling's (1931) T^2 test and has been extensively studied for its optimality properties (Anderson 1958) and used in diverse applications (Kshirsagar 1972; Morrison 1967; Rao 1973). However, some of the properties which make Hotelling's T^2 elegant and simple may render it inappropriate in some applications. For example, the "invariant" T^2-test treats the variables symmetrically and consequently may be unsuitable if these variables have unequal importance in the investigation. Yet the alternatives to the T^2-test are very few. The purpose of this article is to study one such alternative, namely the step-down procedure, and compare it with Hotelling's T^2 solution.

The step-down reasoning for solving multivariate hypothesis testing problems was formally initiated by Roy and Bargmann (1958) in the context of testing multiple independence. J. Roy (1958) extended it to the problem of testing a multivariate general linear hypothesis, which includes Hotelling's problem as a particular case. The step-down solutions assume an a priori ordering of the variables and involve a sequence of tests of significance. The practical importance of the step-down procedure also lies in the fact that it permits recognition of the unequal relevance of the variables in that they can be ordered according to their importance and the overall type I error probability can be distributed suitably between the component tests. From the theoretical standpoint however very little is known about the step-down tests.

In Section 2, we outline the step-down procedure for Hotelling's problem and describe an invariance structure in which this procedure as well as the T^2-test are invariant. In Section 3, some properties of step-down procedure for Hotelling's problem are discussed. Some estimates of the power function of the step-down procedure based on a Monte Carlo experiment and conclusions are presented in the final section.

2. Invariance of the step-down procedure for Hotelling's T^2 problem

In canonical form Hotelling's problem is that of testing a null hypothesis H_0: $\underset{\sim}{\mu} = \underset{\sim}{0}$ for a p-variate normal population with mean vector $\underset{\sim}{\mu} = (\mu_1, \mu_2, \ldots, \mu_p)'$ and positive definite covariance matrix $\underset{\sim}{\Sigma}$, on the basis of a random sample $\underset{\sim}{X}_i = (x_{i1}, \ldots, x_{ip})'$, $i = 1, 2, \ldots, n$, $n > p$. If $\underset{\sim}{\bar{X}} = \sum_{i=1}^{n} \underset{\sim}{X}_i/n$ and $\underset{\sim}{S} = \sum_{i=1}^{n} (\underset{\sim}{X}_i - \underset{\sim}{\bar{X}})(\underset{\sim}{X}_i - \underset{\sim}{\bar{X}})'$, then Hotelling's test rejects H_0 for large values of $T^2 = n(n-1)\underset{\sim}{\bar{X}}' \underset{\sim}{S}^{-1}\underset{\sim}{\bar{X}}$, where the critical constant for the test is determined by the fact that under H_0, $(n-p)T^2/[p(n-1)]$ has an F-distribution with p and $(n-p)$ degrees of freedom.

The step-down procedure proposed by J. Roy (1958) for the MANOVA problem, when particularized for testing H_0: $\underset{\sim}{\mu} = 0$, consists of a sequence of tests based upon statistics

$$F_i = \frac{(n-i)(T_i^2 - T_{i-1}^2)}{[(n-1)+T_{i-1}^2]}, \quad i = 1, \ldots, p, \qquad (2.1)$$

where $T_0^2 = 0$, and T_i^2 denotes Hotelling's T^2 statistic for testing $(\mu_1, \ldots, \mu_i) = 0'$ based upon the first i variates, $i = 1,2,\ldots p$. The null hypothesis H_0 is rejected as soon as a component test in the sequence shows significance. It is well known (J. Roy 1958; Roy, Gnanadesikan and Srivastava 1971 [p. 473]) that under H_0, the F_i's are independently distributed according to F distributions with degrees of freedom 1 and $(n-i)$. Consequently the size α of the overall procedure is related to the levels α_i of the component tests by $(1-\alpha) = \prod_{i=1}^{p} (1-\alpha_i)$.

The well known invariance structure underlying the MANOVA problem (e.g., Lehmann 1959; Ferguson 1967) involves the following three groups of transformations: (i) the translation group which removes the irrelevant observations from consideration, (ii) the orthogonal groups which reduce the data to the sum of squares and product matrices H and E due to hypothesis and due to error respectively, and (iii) the group of premultiplication by nonsingular matrices, which finally yields the eigenvalues of HE^{-1} as a set of maximal invariants. In this invariance fromework the T^2 statistic is maximal invariant for Hotelling's problem and the T^2-test is best invariant procedure (lehmann 1959, [p. 299]). Alternatively, sufficiency considerations, instead of the group in (ii), also lead to H and E. In the T^2-case one could dispense with the groups at both the first and the second stages. Thus among all procedures for H_0 based upon the sufficient statistics $\bar{x}$ and S which are invariant under transformations $\bar{x}^* = C\bar{x}$ and $S^* = CSC'$, where C is a non-singular matrix, the T^2 test is known to be UMP (Anderson 1958, [Thoerem 5.5.1, p. 115]). Now we show that the step-down statistics are maximal invariants when the transformation matrices C are restricted to be lower triangular (Theorem 2.1), and later discuss the nature of the nonnull joint-distribution of these statistics (Thoerem 2.2).

<u>Theorem 2.1</u>. If T_i^2 denotes Hotelling's T^2 statistic for testing $\mu_i = (\mu_1,\mu_2,\ldots,\mu_i)' = 0$ based on the first i variates, $i = 1,2,\ldots p$, where $\mu_1,\ldots,\mu_p$ are the means of the p-variables, then the step-down statistics in (2.1) are maximal invariants of sufficient statistic $(\bar{x}, S)$ under the group of transformations $\bar{x}^* = L\bar{x}$, $S^* = LSL'$, where L is a nonsingular lower triangular matrix.

<u>Proof</u>. The statistics $F_1,F_2,\ldots,F_p$ are invariant, since $T_1^2,T_2^2,\ldots,T_p^2$ remain invariant under nonsingular lower triangular transformations. We now show that they are maximal invariants. Suppose that two sets of data $\bar{x},S_x$ and $\bar{y},S_y$

give rise to the same step-down statistics. Let L_x and L_y denote lower triangular matirices such that $S_x = L_x L_x'$ and $S_y = L_y L_y'$, and $u = L_x^{-1}\bar{x} = (u_1,\ldots,u_p)'$, $v = L_y^{-1}\bar{y} = (v_1,\ldots,v_p)'$. Since both sets of data yield the same step-down statistics, we have $\sum_{j=1}^{i} u_j^2 = \sum_{j=1}^{i} v_j^2$, $i = 1,\ldots,p$, or equivalently $u_i^2 = v_i^2$, i.e., $u_i = \pm v_i$, $i = 1,\ldots p$. Hence $v = L_y^{-1}\bar{y} = Mu = ML_x^{-1}\bar{x}$, where M is a diagonal matrix with either $+1$ or -1 as the diagonal elements. By taking $L = L_y M L_x^{-1}$, we find that $\bar{y} = L\bar{x}$ and $S_y = LS_x L'$, thus completing the proof of the theorem.

Theorem 2.2. Maximal invariants in the parameter space, under the induced group of transformations, are $\mu_i' \Sigma_i^{-1} \mu_i$, $i = 1,\ldots p$. Equivalently, if B is the lower triangular matrix such that $\Sigma = BB'$, and $\eta = B^{-1}\mu = (\eta_1, \eta_2, \ldots, \eta_p)'$, then maximal invariants are $\eta_1^2, \eta_2^2, \ldots, \eta_p^2$.

Proof. The Theorem follows from the above Theorem 2.1, by replacing $\bar{x}$ by μ, and S by Σ.

Theorem 2.2 implies that the joint distribution of the step-down statistics involves only η_i^2, $i = 1,\ldots,p$, as the non-centrality parameters, and that the power function of the step-down procedure for Hotelling's problem depends on these non-centrality parameters only (Lehmann 1959, [p. 220, Theorem 3]). Clearly, the T^2 test is also invariant under the transformations considered in Theorem 2.1. Its power depends on $\eta_1^2,\ldots,\eta_p^2$ only through $\sum_{i=1}^{p} \eta_i^2$.

Remark 2.1. In the invariance framework, described above, for the MANOVA problem if the group in (iii) is replaced by the group of nonsingular lower triangular matrices, then it can be shown that the step-down statistics are invariant. However, they do not appear to be maximal.

3. Some properties of the step-down procedure

In this section we study the unbiasedness and monotonicity of the power function of the step-down procedure.

If $F_1,\ldots,F_p$ are the step-down statistics and $\alpha_1,\ldots,\alpha_p$ are the levels of significance with corresponding critical constants $c_1,\ldots,c_p$, then the power function is given by

$$\beta = 1 - \int_{-\infty}^{c_1} \cdots \int_{-\infty}^{c_p} g_1(F_1) g_2(F_2|F_1) \cdots g_p(F_p| F_1,\ldots,F_{p-1}) dF_1 \ldots dF_p \quad (3.1)$$

where $g_i(F_i|F_1,\ldots,F_{i-1})$ denotes the conditional distribution of F_i given $F_1,\ldots,F_{i-1}$ under the alternative hypothesis at which β is calculated. Then, we have the following theorem.

Theorem 3.1. The step-down procedure is unbiased, if the component tests are unbiased.

Proof. Given that the component tests are unbiased, we have

$\int_{-\infty}^{c_i} g_i(F_i|F_1,\ldots,F_{i-1}) dF_i \leq (1-\alpha_i)$ for $i = 1,2,\ldots,p$ and consequently

$$(1-\alpha) = \prod_i (1-\alpha_i) \geq \prod_{i=1}^{p} \int_{-\infty}^{c_i} g_i(F_i|F_1,\ldots,F_{i-1}) dF_i = 1 - \beta . \quad \square$$

Remark 3.1. Theorem 3.1 holds for the step-down procedure associated with MANOVA also, where the step-down statistics are independently distributed under the null hypothesis.

Theorem 3.2. If the component tests are consistent (i.e., the power increases to one as $n \to \infty$), then the step-down procedure is consistent for any fixed alternative hypothesis.

Proof. From (3.1) we have $1 - \beta = \prod_{i=1}^{p} \gamma_i(n)$, where $\gamma_i(n)$ is the probability of type II error of i^{th} component test. Since the component tests are consistent, $\gamma_i(n) \to 0$ as $n \to \infty$, and consequently $\beta \to 1$. $\square$

We now consider the case $p = 2$ for simplicity and discuss some results related to the power of the step-down procedure.

Theorem 3.3. The power function of the step-down procedure for testing H_0: $(\mu_1,\mu_2) = (0,0)$ is an increasing function of η_1^2 if $\eta_2^2 = 0$, and an increasing function of η_2^2 when η_1^2 is fixed.

Proof. By (2.1), F_2 has conditionally a non-central F distribution with d.f. (1,n-2) and the non-centrality parameter $\eta_2^2/[(n-1) + F_1]$, it follows that

$$I(c_2,\eta_2^2/[(n-1) + F_1]) = \int_0^{c_2} g_2(F_2|F_1)dF_2 \qquad (3.2)$$

is a decreasing function of η_2^2, and increasing function of F_1. Similarly, $\int_0^{c_1} g_1(F_1)dF_1$ is a decreasing function of η_1^2.

It follows easily that if $\eta_2 = 0$, $I(c_2,\eta_2^2/[(n-1) + F_1])$ is independent of η_1^2 and consequently (3.1) is an increasing function of η_1^2. For a fixed value of η_1, $I(c_2,\eta_2^2/[(n-1) + F_1])$ is a decreasing function of η_2^2 implying that (3.1) also is an increasing function of η_2^2. □

For general p, the above theorem may be extended as follows:

Theorem 3.4. The power function of the step-down procedure for testing the hypothesis $\underset{\sim}{\mu} = \underset{\sim}{0}$, is an increasing function of η_i^2 when $\eta_1^2,\dots,\eta_{i-1}^2$ are fixed and $\eta_{i+1}^2 = \dots = \eta_p^2 = 0$, $i = 1,\dots,p$.

The proof is similar to that of Theorem 3.3.

In order to see whether the power function of the step-down procedure is an increasing function of η_1^2 when η_2^2 is fixed, we may proceed as follows: If we denote,

$$I(x) = \begin{cases} \int_0^{c_2} g_2(F_2|F_1 = x)dF_2 & \text{if} \quad x \le c_1 \\ 0\text{, otherwise} & \end{cases}$$

then $E_{\eta_1^2}(I(x)) = 1-\beta(\eta_1^2)$ as given in (3.1). Let η_1' and η_1'' be two values of η_1 such that $|\eta_1'| < |\eta_1''|$, and $g_1(F_1;\eta_1'),g_1(F_1;\eta_1'')$ denote the p.d.f. of F_1 when $\eta_1 = \eta_1'$, and $\eta_1 = \eta_1''$ respectively. Since the distribution of F_1 belongs to montone likelihood ratio family (Lehmann [5], p. 68), there exists a point d such that $g_1(x;\eta_1') > g_1(x;\eta_1'')$ if $x \le d$, and $g_1(x;\eta_1') < g_1(x;\eta_1'')$ if $x > d$. But,

$$
\begin{aligned}
E_{\eta_1''}(I(x)) - E_{\eta_1'}(I(x)) &= \int_0^\infty I(x)\,[g_1(x;\eta_1'')-g_1(x;\eta_1')]\,dx \\
&= \int_d^\infty I(x)\,[g_1(x;\eta_1'')-g_1(x;\eta_1')]\,dx \qquad (3.3)\\
&\quad - \int_0^d I(x)\,[g_1(x;\eta_1')-g_1(x;\eta_1'')]\,dx
\end{aligned}
$$

If $c_1 < d$, then the first term in the R.H.S. of (3.3) is zero, for which reason $E_{\eta_1''}(I(x)) < E_{\eta_1'}(I(x))$ and the power at η_1'' will be larger than that at η_1'. For $c_1 > d$, (3.3) could be positive, in which case the power at η_1'' will be smaller than the power at η_1'.

4. A study of the power function by simulation

In this section we describe a sampling experiment which provides an estimate of the power function of the step-down procedure (of Section 2) for $p = 2$. Without any loss of generality we take $\underset{\sim}{\Sigma} = \underset{\sim}{I}_2$, the identity matrix of order 2, in which case the power of the invariant procedures depends upon $\eta_1^2 = \mu_1^2$, and $\eta_2^2 = \mu_2^2$. Consequently we can restrict our attention to non-negative values of μ_1, μ_2 and observe the behavior of the power as μ_1 and μ_2 change.

The Monte Carlo experiment: The objective of the experiment is to investigate the power function of the step-down procedure and compare it with the power function of Hotelling's T^2 procedure. The power of these procedures has been computed for several values of μ_1, μ_2, namely $\mu_1 = 0.0\ (0.1)\ 1.6$, and $\mu_2 = 0.0\ (0.1)\ 1.9$.

The standard normal deviates are generated on the IBM 360/365 computer at the University of Rochester using "McGill University random number package" based upon the technique of Marsaglia [6] for generating standard normal deviates. A random observation from the bivariate normal population $N_2(\mu, I_2)$ is obtained by drawing two random observations from a standard univariate normal population and adding μ_1 and μ_2 to them respectively.

Test procedures considered in the simulation experiment: For a fixed value of (μ_1, μ_2), the power is estimated on the basis of 3000 samples of size 20, corresponding to two values of the level of significance α, namely .01 and .05 . After each sample of size 20 is drawn, the data are subjected to the following tests of significance, each of the tests being at the two values of α.

The step-down procedure is applied by setting different values for the levels of significance α_1, α_2 corresponding to the two component tests, such that

$$(1-\alpha) = (1-\varepsilon)^{r_1+r_2}, \quad (1-\alpha_i) = (1-\varepsilon)^{r_i}, \quad i = 1,2. \tag{4.1}$$

The correspondence between α_i's and r_i's is presented in Table 1, with (r_1, r_2) taking values (10,1),(4,1),(2,1),(1,1),(1,2),(1,4),(1,10) and $\alpha = .01$ and .05 . The power of a particular procedure at a given value of $\underset{\sim}{\mu}$ is estimated

(TABLE 1 TO GO HERE)

by the proportion of times the test rejects H_0 in the 3000 trials.

The power of Hotelling's procedure depends upon $\mu_1^2 + \mu_2^2$, which is symmetric in μ_1^2 and μ_2^2. Hence the power is obtained for $\mu_1 \geq \mu_2$ using the routine by Bargmann and Ghosh (1964) for calculating the cdf of a noncentral F-distribution with (2, 18) d.f. and is presented in Table 2 corresponding to $\alpha = .01$, and .05

(TABLE 2 TO GO HERE)

<u>Results:</u> Several conclusions may be drawn from Table 2 which presents the exact power function of Hotelling's T^2 based on noncentral F distribution and Tables 3-8 which present some of the estimates of power function of the step-down procedure based on simulation study.

(1) When $r_1 = r_2$, i.e., $\alpha_1 = \alpha_2$ the simulation indicates a slight superiority of the step-down procedure over the T^2-test along the corrdinate axes i.e., $\mu_1 = 0$ or $\mu_2 = 0$. On the other hand T^2-test dominates the step-down procedure along the equiangular line $\mu_1 = \mu_2$.

(2) As observed in Thoerem 3.3, the power of the step-down procedure

corresponding to each value of (r_1, r_2), is an increasing function of μ_1 if $\mu_2 = 0$, and an increasing function of μ_2 when μ_1 is fixed, with only a few insignificant exceptions.

(3) The power of the step-down procedure at (μ_1, μ_2) appears to be an increasing function of α_1 if $\mu_1 > \mu_2$, and a decreasing function of α_1 if $\mu_1 < \mu_2$. Note that for a fixed α, α_1 may be increased either by increasing r_1 for a fixed value of r_2, or by decreasing r_2 for a fixed value of r_1. (See Table 1).

(4) When $\mu_1 \neq \mu_2$, a selection of (r_1, r_2) seems possible such that the power of the step-down procedure at (μ_1, μ_2) is larger than that of Hotelling's T^2 procedure. But when $\mu_1 = \mu_2$, such a selection of (r_1, r_2) is not possible.

(5) The power of the step-down procedure does not seem to be an increasing function of μ_1 in the range 0.0 to 1.6, for every fixed value of $\mu_2 > 0$.

Conclusion: From the results of the simulation study it may be concluded that if there is an a priori ordering among the response variables then the step-down procedure may be used in place of Hotelling's T^2-test. If the levels of the component tests are equal then the power function of the step-down procedure is not very different from that of T^2-test. But by taking the level of the first component test α_1 large, i.e., r_1 large, the power of the step-down method in detecting the significance of μ_1 can be substantially increased over the corresponding power of the T^2-test.

This conclusion is supported by our earlier work on the multiple comparisons associated with the step-down procedure (Mudholkar and Subbaiah 1975, 1976). There we observed that the confidence intervals for the means of the variables appearing earlier in the step-down sequence are shorter than the corresponding widths associated with the T^2-test, or the largest root test in case of MANOVA.

In summary, when the variables in a multiresponse experiment are of unequal practical significance, and are ordered accordingly, a step-down analysis seems to yield superior inferences on the earlier variables at the expense of the quality of the inferences on the later variables, as compared with the corresponding inferences obtained using conventional methods such as Hotelling's T^2.

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1. CORRESPONDENCE BETWEEN r_i's AND α_i's

		sets of (r_1,r_2)						
		(10,1)	(4,1)	(2,1)	(1,1)	(1,2)	(1,4)	(1,10)
$\alpha=.01$	α_1	.0091	.0080	.0067	.0050	.0034	.0020	.0009
	α_2	.0009	.0020	.0034	.0050	.0067	.0080	.0091
$\alpha=.05$	α_1	.0456	.0402	.0336	.0253	.0170	.0102	.0046
	α_2	.0046	.0102	.0170	.0253	.0336	.0402	.0456

2. EXACT POWER FUNCTION OF HOTELLING'S T^2 TEST PROCEDURE[a]

α = .01 and .05

μ_2								
1.3	.999	.999	.999	1.00	1.00	1.00	1.00 1.00	1.00
1.0	.965	.966	.976	.989	.996	1.00 .995	1.00	1.00
0.7	.732	.742	.806	.895	.962 .837	.968	.998	1.00
0.5	.436	.451	.564	.742 .466	.689	.929	.994	.999
0.3	.181	.197	.326 .128	.291	.548	.882	.989	.999
0.1	.063	.076 .018	.063	.206	.466	.851	.985	.998
0.0	.050 .010	.014	.056	.195	.456	.846	.984	.998
	0.0	0.1	0.3	0.5	0.7	1.0	1.3	1.5

μ_1

a: Below diagonal elements correspond to α = .01 and above diagonal elements correspond to α = .05 .

3. THE POWER FUNCTION OF THE STEP-DOWN PROCEDURE ESTIMATED FROM THE MONTE CARLO EXPERIMENT OF SECTION 4

$\alpha = .01, \quad \underset{\sim}{r} = (10,1)$

μ_2								
1.5	.987	.985	.979	.975	.984	.998	1.00	1.00
1.3	.926	.924	.916	.909	.938	.990	1.00	1.00
1.0	.648	.648	.650	.697	.811	.962	.999	1.00
0.7	.246	.257	.282	.422	.660	.935	.997	.999
0.5	.085	.087	.148	.333	.607	.922	.997	1.00
0.3	.015	.027	.099	.282	.609	.931	.996	1.00
0.1	.013	.021	.092	.260	.580	.930	.995	.999
0.0	.011	.016	.080	.277	.585	.929	.996	.999
μ_1	0.0	0.1	0.3	0.5	0.7	1.0	1.3	1.5

4. THE POWER FUNCTION OF THE STEP-DOWN PROCEDURE ESTIMATED FROM THE MONTE CARLO EXPERIMENT OF SECTION 4

$\alpha = .01, \quad \underset{\sim}{r} = (1,1)$

μ_2								
1.5	.998	.997	.997	.995	.995	.998	1.00	1.00
1.3	.981	.985	.980	.966	.976	.996	.999	1.00
1.0	.863	.860	.843	.835	.878	.965	.998	.999
0.7	.479	.485	.479	.530	.672	.924	.993	.998
0.5	.203	.209	.217	.349	.562	.893	.993	1.00
0.3	.043	.059	.107	.244	.531	.889	.992	.999
0.1	.017	.019	.066	.204	.490	.883	.991	.998
0.0	.011	.010	.059	.208	.495	.885	.990	.999
μ_1	0.0	0.1	0.3	0.5	0.7	1.0	1.3	1.5

5. THE POWER FUNCTION OF THE STEP-DOWN PROCEDURE ESTIMATED FROM THE MONTE CARLO EXPERIMENT OF SECTION 4

$\alpha = .01, \underset{\sim}{r} = (1,10)$

μ_2								
1.5	1.00	1.00	.997	.995	.994	.995	.999	1.00
1.3	.991	.991	.986	.971	.965	.984	.996	.999
1.0	.910	.903	.884	.846	.844	.916	.983	.997
0.7	.566	.571	.540	.527	.557	.805	.957	.993
0.5	.270	.274	.249	.291	.397	.739	.955	.993
0.3	.068	.075	.093	.144	.304	.702	.954	.991
0.1	.017	.015	.028	.085	.273	.692	.949	.999
0.0	.009	.008	.024	.079	.275	.708	.943	.989
	0.0	0.1	0.3	0.5	0.7	1.0	1.3	1.5

μ_1

6. THE POWER FUNCTION OF THE STEP-DOWN PROCEDURE ESTIMATED FROM THE MONTE CARLO EXPERIMENT OF SECTION 4

$\alpha = .05, \underset{\sim}{r} = (10,1)$

μ_2							
1.3	.983	.986	.988	.987	.995	1.00	1.00
1.0	.868	.870	.884	.917	.968	.997	1.00
0.7	.494	.510	.577	.741	.898	.990	1.00
0.5	.227	.252	.374	.636	.860	.987	1.00
0.3	.076	.102	.279	.554	.845	.990	.999
0.1	.055	.082	.239	.531	.832	.986	1.00
0.0	.051	.070	.232	.548	.831	.987	1.00
	0.0	0.1	0.3	0.5	0.7	1.0	1.3

μ_1

7. THE POWER FUNCTION OF THE STEP-DOWN PROCEDURE EXTIMATED FROM THE MONTE CARLO EXPERIMENT OF SECTION 4

$\alpha = .05, \quad \underset{\sim}{r} = (1,1)$

μ_2 \ μ_1	0.0	0.1	0.3	0.5	0.7	1.0	1.3
1.3	.998	.999	.999	.997	.999	1.00	1.00
1.0	.996	.969	.971	.969	.985	.998	1.00
0.7	.749	.749	.771	.831	.914	.986	1.00
0.5	.441	.446	.507	.674	.843	.981	1.00
0.3	.163	.179	.295	.507	.786	.981	.999
0.1	.063	.078	.197	.435	.757	.974	.999
0.0	.052	.060	.182	.457	.757	.975	1.00

8. THE POWER FUNCTION OF THE STEP-DOWN PROCEDURE ESTIMATED FROM THE MONTE CARLO EXPERIMENT OF SECTION 4

$\alpha = .05, \quad \underset{\sim}{r} = (1,10)$

μ_2 \ μ_1	0.0	0.1	0.3	0.5	0.7	1.0	1.3
1.3	.999	1.00	.999	.997	.999	1.00	1.00
1.0	.979	.982	.977	.966	.974	.993	1.00
0.7	.815	.825	.809	.814	.851	.965	.997
0.5	.530	.529	.527	.589	.710	.927	.996
0.3	.217	.220	.255	.354	.591	.899	.992
0.1	.065	.074	.116	.243	.511	.886	.992
0.0	.047	.045	.091	.228	.505	.882	.989

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Hotelling's T^2 procedure for testing the significance of a mean vector treats all variables symmetrically and may not be appropriate if the variables carry unequal importance. The step-down procedure, due to J. Roy (1958), a possibly appropriate alternative, is studied in this paper. An underlying invariance structure is established and then used to develop a canonical form for studying the power functions of the two methods. A Monte Carlo experiment is conducted in this framework and conclusions are reported.

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